Reg.No.:

VIVEKANANDHA COLLEGE OF ENGINEERING FOR WOMEN
[AUTONOMOUS INSTITUTION AFFILIATED TO ANNA UNIVERSITY, CHENNAI]
Elayampalayam – 637 205, Tiruchengode, Namakkal Dt., Tamil Nadu.

Question Paper Code: 20008

B.E. / B.Tech. DEGREE END-SEMESTER EXAMINATIONS – JAN. 2025
Fourth Semester
Electronics and Communication Engineering
U19MA407- PROBABILITY AND RANDOM PROCESSES
(Regulation 2019)

Time : Three Hours Maximum : 100 Marks

Answer ALL the questions

Knowledge Levels (KL)	K1 – Remembering	K3 – Applying	K5 - Evaluating
	K2 – Understanding	K4 – Analyzing	K6 - Creating

PART – A

(10 x 2 = 20 Marks)

<table>
<tr><th>Q.No.</th><th>Questions</th><th>Marks</th><th>KL</th><th>CO</th></tr>
<tr><td>1.</td><td>A random variable X has the following probability function
<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>P(X)</td><td>0.1</td><td>0.2</td><td>K</td><td>2K</td><td>0.1</td></tr></table>Find the value of K.</td><td>2</td><td>K1</td><td>CO1</td></tr>
<tr><td>2.</td><td>Find the MGF of the RV X whose PDF $P(X=x)=\frac{1}{2^x}, x=1,2,3....$</td><td>2</td><td>K1</td><td>CO1</td></tr>
<tr><td>3.</td><td>If the joint PDF of the RV (X, Y) is given by $f(x,y)=\begin{cases} e^{-(x+y)}, & 0<x,y<\infty \\ 0 & ,otherwise \end{cases}$ Are X and Y independent?</td><td>2</td><td>K2</td><td>CO2</td></tr>
<tr><td>4.</td><td>The join PDF of a random variable (x,y) is $f(x,y)= xy^2+x^2/8$, $0\leq x\leq 2$, $0\leq y\leq 1$ Find $P(x<y)$</td><td>2</td><td>K2</td><td>CO2</td></tr>
<tr><td>5.</td><td>Distinguish between WSS & SSS.</td><td>2</td><td>K1</td><td>CO3</td></tr>
<tr><td>6.</td><td>Define Gaussian process.</td><td>2</td><td>K2</td><td>CO3</td></tr>
<tr><td>7.</td><td>Check whether the function $\frac{1}{1+4\tau^2}$ is valid auto correlation function.</td><td>2</td><td>K2</td><td>CO4</td></tr>
<tr><td>8.</td><td>Write properties of cross power spectral density.</td><td>2</td><td>K2</td><td>CO4</td></tr>
<tr><td>9.</td><td>Define a linear system.</td><td>2</td><td>K1</td><td>CO5</td></tr>
</table>

10. Find the auto correlation function of Gaussian white noise. 2 K2 CO5

PART – B

(5 x 16 = 80 Marks)

Q.No. Questions Marks KL CO

11. a) i. A RV X has the following distribution 8 K3 CO1

x	0	1	2	3	4	5	6	7
P(X)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

a. Find the value of ‘k’

b. Find $P(X < 6)$, $P(X \geq 6)$ & $P(< X < 5)$

c. If $P(X \geq k) > \frac{1}{2}$ find the minimum value of ‘k’

d. Find the distribution function of x.

ii. Obtain MGF, mean and variance of Poisson distribution. 8

(OR)

b) i. In a large consignment of electric bulbs 10 % are defective. A random sample of 20 bulbs is taken for inspection. Find the probability that 8 K3 CO1

a. all are good bulbs

b. At most there are three defective bulbs

c. exactly three defective bulbs.

ii. The mileage which car owners get with a certain kind of radial type is RV having an exponential distribution with mean 40,000 km. find the probabilities that one these tires will last 8

a. At least 20,000 km

b. at most 30,000 km.

12. a) i. The joint pdf $f(x, y) = k\,x\,y\,e^{-(x^2 + y^2)}$, $0 < x, y < \infty$. Find the value of K and 8 K3 CO2

a. Show that X and Y are independent

b. conditional density function of X given Y.

ii. Find the covariance of X and Y if the random variables (X,Y) having joint probability density function f(x,y)={$x + y; 0 \leq x \leq 1, 0 \leq y \leq 1$ 8

(OR)

b) From the following data Find 16 K3 CO2

i. Two regression equations.

ii. The coefficient of correlation between the marks in Economics and Statistics.

iii. The most likely marks in Statistics in the marks in Economics are 30.

Marks in Economics	25	28	35	32	31	36	29	38	34	32
Marks in Statistics	43	46	49	41	36	32	31	30	33	39

13. a) i. The transition probability matrix of the Markov chain $\{X_n\}$, n = 1,2,3…..having three states 1, 2 and 3 is 8 K3 CO3

$$\begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$ and the initial state distribution is

$$P^{(0)} = (0.7 \quad 0.2 \quad 0.1)$$

Find

$a) P(X_2 = 3)$ $\quad b) P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$

ii. Show that the random process $X(t) = A\cos(\omega t + \theta)$ is Wide Sense Stationary if A and ω are constant and θ is uniformly distributed in $(-\pi, \pi)$. 8

(OR)

b) i. Prove that the random process $X(t) = A\cos(\omega t + \theta)$, A and ω are constants and θ is uniformly distributed random variable in (0,2π) is mean ergodic. 8 K3 CO3

ii. If customers arrive at a counter in accordance with a poisson process with mean rate of 3 per minute find the probability that interval between 2 consecutive arrivals is 8

a. more than one minutes

b. between 1 minute and 2 minutes

c. 4 minutes or less.

14. a) i. Find power spectral density of a WSS process whose auto correlation function $R_{XX}(\tau) = e^{-\alpha\tau^2}, \alpha > 0$. 8 K3 CO4

ii. If {X(t)} and {Y(t)} are two random process and $X(t) = A\cos(\omega t + \theta)$ and $Y(t) = Bsin(\omega t + \theta)$ where A, B and ω are constants and θ is a uniform random variable ever $(0, 2\pi)$ then find the cross correlation function of {X(t)} and {Y(t)} 8

(OR)

b) i. If {X(t)} is WSS with Auto correlation $R_{XX}(\tau)$ and $Y(t) = X(t+a) - X(t-a)$ 8 K3 CO4

Prove that

$$R_{YY}(\tau) = 2R_{XX}(\tau) - R_{XX}(\tau + 2a) - R_{XX}(\tau - 2a)$$

ii. Find cross power spectrum if the cross correlation of two processes {X(t)} and {Y(t)} 8

is $R_{XY}(t, t+\tau) = \frac{AB}{2}[\sin(\omega\tau) + \cos(\omega(2t+\tau))]$,

where A, B and ω are constants.

15. a) 16 K5 CO5

If {X(t)} is a WSS process and if $Y(t) = \int_{-\infty}^{\infty} h(u)X(t-u)du$

then Prove that

i. $R_{xy}(\tau) = R_{xx}(\tau)\, h(\tau)$
ii. $R_{yy}(\tau) = R_{xy}(\tau)\, h(-\tau)$
iii. $S_{xy}(\omega) = S_{xx}(\omega)\, H(\omega)$
iv. $S_{yy}(\omega) = S_{yx}(\omega) |H(\omega)|^2$

(OR)

b) i. If {X(t)} is a band limited process such that $S_{XX}(\omega) = 0$ when 8 K5 CO5

$|\omega| > \sigma$, *prove that* $2\left[R_{XX}(0) - R_{XX}(\tau)\right] \leq \sigma^2\tau^2 R_{XX}(0)$

ii. If $Y(t) = A\cos(\omega_0 t + \theta) + N(t)$ where A is a constant, θ is Random variable with uniform distribution in $(-\pi, \pi)$ and{N(t)} is a band limit Gaussian white noise with a power spectral density 8

$$S_{NN}(\omega) = \begin{cases} \frac{N_0}{2} & , for\ |\omega - \omega_0| < \omega_B \\ 0 & , else\,where \end{cases}$$

.Find the power spectral density of {Y(t)} assume that N(t) and θ are independent.